

Interference Channel with a Half-Duplex Out-of-Band Relay

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Abstract—A Gaussian interference channel (IC) aided by a half-duplex relay is considered, in which the relay receives and transmits in an orthogonal band with respect to the IC. The system thus consists of two parallel channels, the IC and the channel over which the relay is active, which is referred to as Out-of-Band Relay Channel (OBRC). The OBRC is operated by separating a multiple access phase from the sources to the relay and a broadcast phase from the relay to the destinations. Conditions under which the optimal operation, in terms of the sum-capacity, entails either *signal relaying* and/or *interference forwarding* by the relay are identified. These conditions also assess the optimality of either separable or non-separable transmission over the IC and OBRC. Specifically, the optimality of signal relaying and separable coding is established for scenarios where the relay-to-destination channels set the performance bottleneck with respect to the source-to-relay channels on the OBRC. Optimality of interference forwarding and non-separable operation is also established in special cases.

I. INTRODUCTION

Consider two interfering links, say belonging to a Wireless Local Area Network, that operate over the same bandwidth, i.e., an interference channel (IC). The corresponding transmitters and receivers of the IC may be also endowed with a second, shorter-range, radio interface, such as Bluetooth, that can be used for communications with an external terminal over an orthogonal bandwidth, as shown in Fig. 1. This terminal may act as a relay for both links, while operating out-of-band with respect to the IC (i.e., as an Out-of-Band Relay, or OBR). By this means, communication takes place effectively over two parallel channels, the IC and the channel where the relay is active, which is termed as OBR channel (OBRC). We refer to the overall channel comprising IC and OBRC as IC-OBR. This scenario was first considered in [1], where it was assumed that the OBRC is operated via an orthogonal medium access scheme (e.g., TDMA) that makes the links from each transmitter to the relay, and from the relay to each receiver, all orthogonal to one another. In this paper, we study the more complex situation in which the relay is simply assumed to be half-duplex, so that the OBRC is operated by allowing the relay to either transmit or receive at a given time.

The considered model is related to, and inspired by, two recent lines of work. The first deals with *relaying in interference-limited systems*, where, unlike in the IC-OBR, the relay is assumed to operate in the same band as the IC [2]–[6][1]. These works

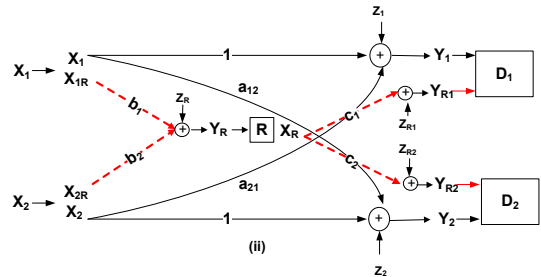


Fig. 1. Interference Channel (IC) with an out-of-band relay (OBR).

reveal the fact that relaying in interference-limited systems offers performance benefits not only due to *signal relaying*, as for standard relay channels, but also thanks to the novel idea of *interference forwarding*. According to the latter, the relay may help by reinforcing the interference received at the undesired destination, so as to facilitate interference stripping, exploiting the standard technique of rate splitting into *private* and *common* messages (see, e.g., [11]): Private messages are decoded only at the intended destination, while common messages are decoded also at the interfered destination (and may benefit from interference forwarding).

A second related line of work deals with communications over *parallel ICs* (albeit the considered OBRC is not a conventional IC). As shown in [7]–[10], optimal operation over parallel ICs, unlike scenarios with a single source or destination, typically entails *joint*, rather than *separate*, coding over the parallel channels. In other words, the signals sent over the parallel ICs need to be generally correlated to achieve optimal performance. The question arises as to what type of information, either private or common, should be sent in a correlated fashion over the component ICs. For instance, the original work [7] derives conditions under which correlated transmission of private messages is optimal, [8] considers the optimality of common information transmission, whereas in [10] scenarios are found for which sending both correlated private and common messages is optimal.

In this paper, we study the IC-OBR model and derive conditions under which a separable coding scheme with only signal relaying is sum-rate optimal, and also conditions under which a non-separable coding scheme with both signal relaying and interference forwarding achieve optimal performance. Analytical results are corroborated by numerical examples.

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II. SYSTEM MODEL

The Gaussian IC-OBRC model, shown in Fig. 1, consists of two parallel channels, namely the IC and the OBRC. On the IC, the signal received by destination D_i at time $t = 1, \dots, n$, is given by

$$Y_{i,t} = X_{i,t} + a_{ji}X_{j,t} + Z_{i,t}, \quad (1)$$

where $i, j = 1, 2$ and $i \neq j$, $Z_{i,t}$ are independent unit-power white Gaussian noise sequences, and $X_{i,t}$ are the transmitted sequences over the IC. Due to the half-duplex constraint, the OBRC is orthogonalized into two channels, one being a multiple-access channel (MAC) from S_1 and S_2 to R , with fraction of channel uses η_{MAC} (relay listening), and the other being a broadcast channel (BC) from R to D_1 and D_2 , with fraction of channel uses η_{BC} (relay transmitting). We have $\eta_{MAC} + \eta_{BC} = \eta$, where η is the ratio between the bandwidths (and thus the channel uses) of the OBRC and of the IC. The received signal at the relay R over the OBRC is given by the MAC relationship

$$Y_{R,t} = b_1X_{1R,t} + b_2X_{2R,t} + Z_{R,t} \quad (2)$$

for $t = 1, \dots, \eta_{MAC}n$ ¹; and the signal received at destination D_i over the OBRC is given by the BC relationships

$$Y_{Ri,t} = c_iX_{R,t} + Z_{Ri,t}, \quad (3)$$

for $t = 1, \dots, \eta_{BC}n$, and $i = 1, 2$. We have the power constraints $\frac{1}{n} \sum_{t=1}^n E[X_{i,t}^2] \leq P_i$ on the IC, and $\frac{1}{n} \sum_{t=1}^{\eta_{MAC}n} E[X_{iR,t}^2] \leq P_{iR}$, $\frac{1}{n} \sum_{t=1}^{\eta_{BC}n} E[X_{R,t}^2] \leq P_R$ on the OBRC, $i = 1, 2$. Bandwidth allocations (η_{MAC}, η_{BC}) will be considered as fixed and given throughout the paper, except in Sec. VI.

Encoding and decoding functions, probability of error and achievable rates are defined in the usual way. In particular, encoding at the source S_i produces two sequences², one transmitted on the IC, X_i^n , and one on the OBRC, X_{iR}^n . The relay decides the transmitted codeword $X_R^{\eta_{BC}n}$ based on the signal received from the sources, namely $Y_R^{\eta_{MAC}n}$ (2). Finally, the destination D_i decodes on the basis of the signals received over the IC, Y_i^n (1), and over the OBRC, $Y_{Ri}^{\eta_{BC}n}$ (3).

III. SCENARIO AND ACHIEVABLE STRATEGIES

The model at hand appears too complicated to hope for general conclusions regarding the capacity region. In fact, the sources may employ a number of rate splitting strategies, with independent or correlated transmission of information over the two parallel channels, IC and OBRC, and may deploy either structured or unstructured codes. Moreover, the relay may implement a number of relaying strategies, encompassing regenerative or non-regenerative techniques. It is emphasized that, not only the optimal operation on the IC alone is generally not known [11], but the same also holds for the operation over the OBRC model alone³. Therefore, in this paper, we focus on specific channel gain conditions and suitable achievable strategies. We will show optimality of the considered techniques in a number of special cases of interest.

In particular, we will consider a scenario in which interference towards receiver D_2 is weak, i.e., $a_{12} \leq 1$ and the channel from

the relay to destination D_2 is worse than towards destination D_1 , i.e., $c_1 \geq c_2$. Under these conditions, we consider the performance of strategies whereby transmitter S_1 , interfering on D_2 transmits: (i) on the IC only *private* information, which is then treated as noise by D_2 ; (ii) on the OBRC independent information, thus using *separate* encoding over IC and OBRC. This choice appears to be reasonable in light of the channel conditions mentioned above. In contrast, transmitter S_2 , whose interference may not necessarily be weak, transmits: (i) on the IC with *both private and common* messages; (ii) on the OBRC with the *same common message* plus an additional independent message. Transmitter S_2 thus potentially employs a non-separable coding strategy where the same (common) message is sent over both IC and OBRC. Since this message is common, the operation of the OBR can be classified as *interference forwarding* [1][3]: The relay forwards information about the interference on the IC from S_2 to D_1 . Moreover, we assume that the OBR employs Decode-and-Foward (DF). The main questions of interest are: Under what conditions is the scheme described above optimal? And, when this is the case, under what conditions is separable (rather than the general non-separable) coding at transmitter S_2 optimal?

IV. OUTER BOUNDS

In this section, we give a general outer bound on the capacity region of IC-OBRC.

Proposition 1 (Outer Bound for IC-OBRC): For an IC-OBRC for $a_{12} \leq 1$ and $c_1 \geq c_2$, with given bandwidth allocation (η_{MAC}, η_{BC}) , the capacity region is included in the following region

$$\lim_{n \rightarrow \infty} \text{closure} \left(\bigcup_{p(x_1^n, x_2^n) = p(x_1^n)p(x_2^n), 0 \leq \xi + \bar{\xi} \leq 1} \{(R_1, R_2): \right.$$

$$R_j \leq \frac{1}{n} I(X_j^n; Y_j^n) + \eta_{MAC} \mathcal{C}(b_1^2 P_{1R} + b_2^2 P_{2R}), \quad (4a)$$

$$R_j \leq \frac{1}{n} I(X_j^n; Y_j^n | X_i^n) + \eta_{MAC} \mathcal{C}(b_j^2 P_{jR}), \quad j = 1, 2, \quad i \neq j \quad (4b)$$

$$R_1 \leq \frac{1}{n} I(X_1^n; Y_1^n | X_2^n) + \eta_{BC} \mathcal{C}(c_1^2 \xi P_R) \quad (4c)$$

$$R_2 \leq \frac{1}{n} I(X_2^n; Y_2^n) + \eta_{BC} \mathcal{C} \left(\frac{c_2^2 \bar{\xi} P_R}{1 + c_2^2 \xi P_R} \right) \Bigg\}, \quad (4d)$$

where the union is taken with respect to multi-letter input distributions $p(x_1^n)p(x_2^n)$ that satisfy the power constraints $\frac{1}{n} \sum_{t=1}^n E[X_{i,t}^2] \leq P_i$, $i = 1, 2$, and with respect to parameters ξ , with $0 \leq \xi \leq 1$ and $\bar{\xi} = 1 - \xi$.

Proof: Appendix A.

V. CAPACITY RESULTS

In this section, we consider fixed OBRC bandwidth allocation (η_{MAC}, η_{BC}) and derive two sets of conditions for optimal operation. Under the first, a special case of the strategy described in Sec. III, in which separable coding only (at both transmitters) is employed, is shown to be optimal, while the second set of conditions provides (asymptotic) optimality of the general non-separable technique.

¹We will not denote explicitly the necessary integer rounding-off operation.

² $X^n \triangleq (X_1, \dots, X_n)$.

³A special case of this model is the two-way relay channel, whose capacity is still generally unknown. Note that the OBRC model is significantly more complex than the four orthogonal links considered in [1].

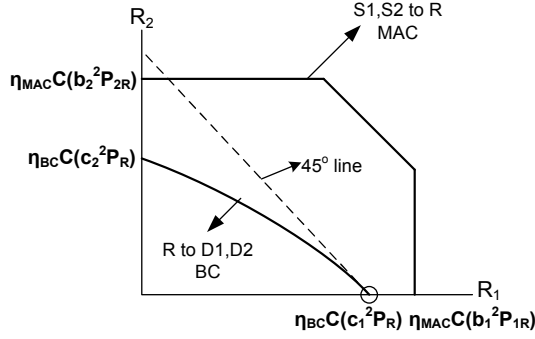


Fig. 2. Illustration of the first OBRC condition leading to the sum-capacity in Proposition 2.

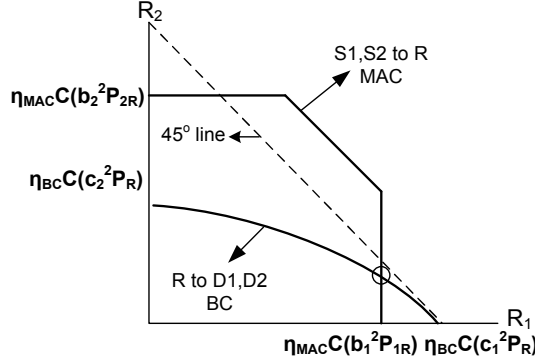


Fig. 3. Illustration of the second OBRC condition leading to the sum-capacity in Proposition 2.

1) Optimality of Separable Encoding:

Proposition 2: In an IC-OBR with $a_{12} \leq 1$, $c_1 \geq c_2$ and $a_{21} \geq \sqrt{\frac{1+P_1}{1+a_{12}^2 P_1}}$, the sum-capacity is given by

$$R_1 + R_2 \leq \mathcal{C}(P_1) + \mathcal{C}\left(\frac{P_2}{1+a_{12}^2 P_1}\right) + \eta_{BC} \mathcal{C}(c_1^2 P_R) \quad (5)$$

if condition $\eta_{MAC} \mathcal{C}(b_1^2 P_{1R}) \geq \eta_{BC} \mathcal{C}(c_1^2 P_R)$ holds; and by

$$R_1 + R_2 \leq \max_{0 \leq \xi \leq 1} \left\{ \mathcal{C}(P_1) + \mathcal{C}\left(\frac{P_2}{1+a_{12}^2 P_1}\right) + \min \left\{ \eta_{MAC} \mathcal{C}(b_1^2 P_{1R}), \eta_{BC} \mathcal{C}(c_1^2 \xi P_R) \right\} + \eta_{BC} \mathcal{C}\left(\frac{c_2^2 \bar{\xi} P_R}{1+c_2^2 \xi P_R}\right) \right\}, \quad (6)$$

if conditions $\eta_{MAC} \mathcal{C}(b_1^2 P_{1R}) < \eta_{BC} \mathcal{C}(c_1^2 P_R)$ and $\eta_{MAC} \mathcal{C}\left(\frac{b_2^2 P_{2R}}{1+b_1^2 P_{1R}}\right) \geq \eta_{BC} \mathcal{C}\left(\frac{c_2^2 \bar{\xi}^* P_R}{1+c_2^2 \xi^* P_R}\right)$ hold, where ξ^* is the optimal power allocation that maximizes the sum-rate (6) with $\bar{\xi}^* = 1 - \xi^*$. The sum-capacity is achieved by separable coding on IC and OBRC.

Proof: The converse follows from Proposition 1 and invoking the worst-case noise result of [12] applied for $a_{12} \leq 1$. For the achievability, we follow the strategy described in Sec. III and we refer to Appendix B for further details. \square

Optimality in Proposition 2 is achieved by using a special case of the strategy described in Sec. III in which transmitter 1 operates as prescribed, and transmitter 2 sends only common information on the IC and transmits independent information over the OBRC (separable coding). Notice that transmission of only common

information over the IC is justified by the “strong interference” condition $a_{21} \geq \sqrt{(1+P_1)/(1+a_{12}^2 P_1)}$. Also, notice that here the relay performs only signal relaying [1]. To be more specific, we have two subcases depending on the channel gains of the OBRC.

The first set of conditions (under which the sum-capacity is (5)) is characterized by $\eta_{MAC} \mathcal{C}(b_1^2 P_{1R}) \geq \eta_{BC} \mathcal{C}(c_1^2 P_R)$ and is illustrated in Fig. 2. It corresponds to the case where the relay-to-destinations BC constitutes the *bottleneck* with respect to the sources-to-relay MAC in the $S_1 - R - D_1$ communication path. In this case, the optimal strategy in terms of sum-rate is for user 1 only to transmit over the OBRC. Notice that this operating point on the OBRC (see dot in the figure) is sum-rate optimal if one focuses on the OBRC alone limiting the scope to DF techniques, since the corresponding achievable rate region is given by the intersection of the MAC and BC regions in Fig. 2. Proposition 2 shows that such operating point is also optimal for communications over the IC-OBR under the given conditions.

The second set of conditions (under which the sum-capacity is (6)) is given by $\eta_{MAC} \mathcal{C}(b_1^2 P_{1R}) < \eta_{BC} \mathcal{C}(c_1^2 P_R)$ and $\eta_{MAC} \mathcal{C}\left(\frac{b_2^2 P_{2R}}{1+b_1^2 P_{1R}}\right) \geq \eta_{BC} \mathcal{C}\left(\frac{c_2^2 \bar{\xi}^* P_R}{1+c_2^2 \xi^* P_R}\right)$, and is illustrated in Fig. 3. Here, the sum-rate optimal operation of the OBRC entails transmission over the OBRC (of independent information) by both sources, not just source 1 as above, at rates given by the operating point indicated in the figure. This rate pair is characterized by the power split ξ^* in Proposition 2. Notice that the same considerations given above regarding optimality of this point for the OBRC alone with DF apply here.

2) *Optimality of Non-Separable Encoding:* In this section, we discuss conditions under which strategies based on interference forwarding, and thus non-separable encoding, are optimal. In Proposition 2, the “strong interference” condition $a_{21} \geq \sqrt{(1+P_1)/(1+a_{12}^2 P_1)}$ at D_1 was instrumental in making interference forwarding from S_2 to D_1 not necessary. Consider then a more general situation in which this condition is not imposed. We will see below that, in this case, interference forwarding (and non-separable coding) is potentially useful. This is shown by first providing an achievable region for a special case of the scheme described in Sec. III, which involves interference forwarding, then establishing its asymptotic optimality under given conditions (that include the scenario discussed above), and finally discussing some numerical results.

Proposition 3: In the IC-OBR, the following conditions

$$R_1 \leq \mathcal{C}(P_1) + \eta_{MAC} \mathcal{C}(b_1^2 P_{1R}) \quad (7a)$$

$$R_2 \leq \mathcal{C}\left(\frac{P_2}{1+a_{12}^2 P_1}\right) + \eta_{BC} \mathcal{C}\left(\frac{c_2^2 \bar{\xi} P_R}{1+c_2^2 \xi P_R}\right) \quad (7b)$$

$$R_1 + R_2 \leq \mathcal{C}(P_1 + a_{21}^2 P_2) + \eta_{BC} \mathcal{C}(c_1^2 \xi P_R) + \eta_{BC} \mathcal{C}\left(\frac{c_2^2 \bar{\xi} P_R}{1+c_2^2 \xi P_R}\right) \quad (7c)$$

$$R_1 + R_2 \leq \mathcal{C}(P_1 + a_{21}^2 P_2) + \eta_{MAC} \mathcal{C}(b_1^2 P_{1R} + b_2^2 P_{2R}) \quad (7d)$$

with $\xi + \bar{\xi} \leq 1$, define a rate region achievable with the scheme of Sec. III. Moreover, for $a_{12} \leq 1$, and $b_2, c_1 \rightarrow \infty$, the sum-capacity is achieved by this scheme and given by

$$R_1 + R_2 \leq \mathcal{C}(P_1) + \mathcal{C}\left(\frac{P_2}{1+a_{12}^2 P_1}\right) + \eta_{MAC} \mathcal{C}(b_1^2 P_{1R}) + \eta_{BC} \mathcal{C}(c_2^2 P_R). \quad (8)$$

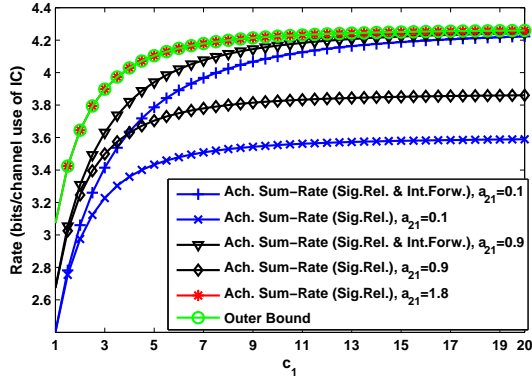


Fig. 4. Achievable sum-rate and outer bound for an IC-OBRC with respect to $R - D_1$ channel gain, c_1 and $S_2 - D_1$ channel gain $a_{21} \in \{0.1, 0.9, 1.8\}$ ($a_{12} = 0.5$, $b_1 = 1$, $b_2 = 10$, $c_2 = 1$ and all node powers are equal to 10).

Proof: Appendix B.

The scheme achieving (7a)-(7d) is based on S_2 transmitting common information over the IC, as for Proposition 2, and both the same common and an independent private message on the OBRC (non-separable encoding). This scheme is shown in Proposition 3 to be sum-rate optimal if a very good channel is available between S_2 and D_1 through the relay, so as to essentially drive D_1 back in the “strong interference regime” thanks to interference forwarding. It is noted that this condition is related to the “large excess rate” assumption of Theorem 4 in [1] (which applies to a TDMA-based operation on the OBRC).

To investigate the role of interference forwarding in a non-asymptotic regime, Fig. 4 shows the sum-rate obtained from (7a)-(7d), by assuming that source 2 either transmits only an independent message on the OBRC (signal relaying, i.e., $R_{2c'} = 0$ in the achievable region given in Appendix B) or also employs interference forwarding, and the sum-rate upper bound obtained from Proposition 1. The OBRC gains are set to $b_1 = 1$, $b_2 = 10$, $c_2 = 1$ and c_1 is varied, all node powers are equal to 10 and $\eta_{MAC} = \eta_{BC} = 1$. We also have $a_{12} = 0.5$ and $a_{21} \in \{0.1, 0.9, 1.8\}$. Note that for $a_{21} = 1.8 \geq \sqrt{(1 + P_1)/(1 + a_{12}^2 P_1)} = 1.78$ the conditions given in Proposition 2 are satisfied and signal relaying alone is optimal. For $a_{21} \in \{0.1, 0.9\} \leq 1.78$, instead, the advantages of interference forwarding become substantial with increasing c_1 , which is due to the fact that the $S_2 - D_1$ pair acquires an increasingly better channel through the relay. Specifically, the asymptotic optimality derived in Proposition 3 is here seen to be in practice attained for finite values of b_2, c_1 .

VI. SOME RESULTS FOR OBRC VARIABLE BANDWIDTH ALLOCATION

We now investigate the effect of being able to optimize the bandwidth allocation (η_{MAC} , η_{BC}) via numerical results. We consider a scenario with $a_{12} = 0.5$, $a_{21} = 1.8$, $c_1 \geq c_2$, and all powers are set to 10, which satisfies the conditions of Prop. 2, except the ones that depend on the bandwidth allocation (η_{MAC} , η_{BC}), which is not specified a priori here. We compare the performance of the achievable scheme of Prop. 2 (separable transmission) with a sum-rate outer bound obtained from Prop. 1. In both cases, the bandwidth allocation (η_{MAC} , η_{BC}) is optimized to maximize the sum-rate. In Fig. 5 (upper part), the sum-rate

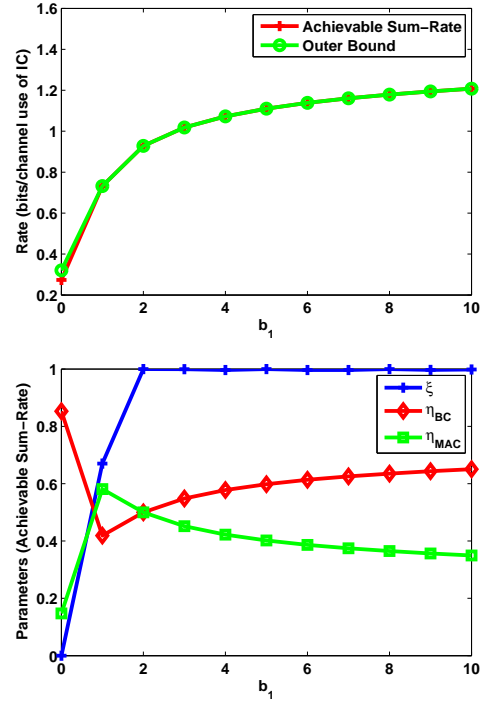


Fig. 5. Achievable sum-rate (from Proposition 2, (5)) with signal relaying and upper bound of Proposition 1 and optimal parameters (η_{MAC} , η_{BC} , ξ) for an IC-OBRC with respect to $S_1 - R$ channel gain b_1 ($b_2 = 2$, $c_1 = 2$, $c_2 = 0.3$, $\eta = 1$, all node powers are equal to 10, $a_{21} = 1.8$, $a_{12} = 0.5$).

discussed above are shown for variable $S_1 - R$ gain, b_1 , and the other channel gains are set to $b_2 = 2$, $c_1 = 2$, $c_2 = 0.3$ and $\eta = 1$. We know from the first part of Prop. 2 that if b_1 is sufficiently larger than c_1 , for fixed bandwidth allocation, the rate (5) where the relay helps the $S_1 - D_1$ pair only, is optimal. Observing the corresponding optimal bandwidth and power allocations for the achievable sum-rate as shown in Fig. 5, a similar conclusion is drawn here for $b_1 \geq 2$ where the achievable sum-rate and outer bound coincide. Moreover, the total bandwidth is balanced between the $S_1 - R$ and $R - D_1$ channels.

VII. CONCLUDING REMARKS

Operation over parallel radio interfaces is bound to become increasingly common in wireless networks due to the large number of multistandard terminals. This enables cooperation among terminals across different bandwidths and possibly standards. In this paper, we have studied one such scenario where two source-destination pairs, interfering over a given bandwidth, cooperate with a relay over an orthogonal spectral resource (out-of-band relaying, OBR). We have derived analytical conditions under which either signal relaying or interference forwarding are optimal. These conditions have also been related to the problem of assessing optimality of either separable or non-separable transmission over parallel interference channels.

APPENDIX

A. PROOF OF PROPOSITION 1

Bounds (4b) follow from cut-set arguments, while (4a) follows as

$$nR_1 \leq H(W_1) \quad (9a)$$

$$\leq I(W_1; Y_1^n, Y_{R1}^{\eta_{BC}n}) + H(W_1|Y_1^n, Y_{R1}^{\eta_{BC}n}) \quad (9b)$$

$$\leq I(W_1; Y_1^n, Y_{R1}^{\eta_{BC}n}) + n\epsilon_n \quad (9c)$$

$$\leq I(X_1^n; Y_1^n) + h(Y_R^{\eta_{MAC}n}) - h(Z_R^{\eta_{MAC}n}) + n\epsilon_n \quad (9d)$$

$$\leq I(X_1^n; Y_1^n) + \eta_{MAC}n\mathcal{C}(b_1^2P_{1R} + b_2^2P_{2R}) + n\epsilon_n \quad (9e)$$

from Fano's inequality, and from the Markov relations $W_1 \rightarrow Y_R^{\eta_{MAC}n}, Y_1^n \rightarrow Y_{R1}^{\eta_{BC}n}$ and $W_1 \rightarrow X_1^n \rightarrow Y_1^n$. Here $\epsilon_n \rightarrow 0$ as $n \rightarrow \infty$. We now focus on the remaining two bound (4c)-(4d), which follow from considerations similar to the Gaussian BC. Proceeding as above, we have

$$\begin{aligned} nR_2 &\leq I(X_2^n; Y_2^n) + h(Y_{R2}^{\eta_{BC}n}) \\ &\quad - h(c_2X_R^{\eta_{BC}n} + Z_{R2}^{\eta_{BC}n}|Y_2^n, W_2) + n\epsilon_n. \end{aligned} \quad (10)$$

Now, consider the following

$$\begin{aligned} h(Z_{R2}^{\eta_{BC}n}) &\leq h(c_2X_R^{\eta_{BC}n} + Z_{R2}^{\eta_{BC}n}|Y_2^n, W_2) \\ &\leq h(c_2X_R^{\eta_{BC}n} + Z_{R2}^{\eta_{BC}n}) \leq \frac{\eta_{BC}n}{2} \log(2\pi e(1 + c_2^2P_R)), \end{aligned}$$

so that, without loss of generality, one can define

$$h(Y_{R2}^{\eta_{BC}n}|Y_2^n, W_2) = \frac{\eta_{BC}n}{2} \log(2\pi e(1 + c_2^2\xi P_R)),$$

for some $0 \leq \xi \leq 1$. Then, (10) becomes

$$nR_2 \leq I(X_2^n; Y_2^n) + \eta_{BC}n\mathcal{C}\left(\frac{c_2^2\xi P_R}{1 + c_2^2\xi P_R}\right) + n\epsilon_n,$$

where we have used the maximum entropy theorem. Now, consider

$$nR_1 \leq I(W_1; Y_1^n, Y_{R1}^{\eta_{BC}n}|W_2) + n\epsilon_n \quad (11a)$$

$$\leq I(X_1^n; Y_1^n|X_2^n) + I(W_1; Y_{R1}^{\eta_{BC}n}|Y_1^n, W_2) + n\epsilon_n \quad (11b)$$

$$= I(X_1^n; Y_1^n|X_2^n) + I(W_1; \frac{c_2}{c_1}Y_{R1}^{\eta_{BC}n}|Y_1^n, W_2) + n\epsilon_n. \quad (11c)$$

Since the capacity region of BC depends on the conditional marginal distributions and noting that $c_1 \geq c_2$, we can write $Y_{R2}^{\eta_{BC}n} = \frac{c_2}{c_1}Y_{R1}^{\eta_{BC}n} + \hat{Z}_R^{\eta_{BC}n}$ where $\hat{Z}_R^{\eta_{BC}n}$ is a Gaussian noise with variance $1 - \frac{c_2^2}{c_1^2}$. From the conditional Entropy Power Inequality, we now have

$$\begin{aligned} 2^{\frac{2}{\eta_{BC}n}h(Y_{R2}^{\eta_{BC}n}|Y_1^n, W_2)} &\geq 2^{\frac{2}{\eta_{BC}n}h(\frac{c_2}{c_1}Y_{R1}^{\eta_{BC}n}|Y_1^n, W_2)} \\ &\quad + 2^{\frac{2}{\eta_{BC}n}h(\hat{Z}_R^{\eta_{BC}n}|Y_1^n, W_2)}. \end{aligned} \quad (12)$$

Also, given that $a_{12} \leq 1$, we have,

$$h(Y_{R2}^{\eta_{BC}n}|Y_1^n, W_2) = h(Y_{R2}^{\eta_{BC}n}|X_1^n + Z_1^n, W_2) \quad (13a)$$

$$= h(Y_{R2}^{\eta_{BC}n}|X_1^n + Z_2^n, W_2) \quad (13b)$$

$$\leq h(Y_{R2}^{\eta_{BC}n}|a_{12}X_1^n + Z_2^n, W_2) \quad (13c)$$

$$= h(Y_{R2}^{\eta_{BC}n}|Y_2^n, W_2) \quad (13d)$$

$$= \frac{\eta_{BC}n}{2} \log(2\pi e(1 + c_2^2\xi P_R)) \quad (13e)$$

due to the Markov chain $a_{12}X_1^n + Z_2^n \rightarrow X_1^n + Z_2^n, W_2 \rightarrow Y_{R2}^{\eta_{BC}n}$. The proof is concluded with standard steps.

B. PROOF OF PROPOSITION 2 AND 3

The achievable region of Prop. 3 is obtained following Sec. III. S_1 transmits a private message W_{1p} over the IC, $X_1^n(W_1) = X_{1p}^n(W_{1p})$, and an independent private message W_{1R} over the OBRC via standard "Gaussian codebooks". S_2 transmits common messages $(W_{2c'}, W_{2c''})$ over the IC ($X_2^n(W_2) = X_{2c}^n(W_{2c'}, W_{2c''})$), and an independent private message W_{2R} , along with $W_{2c'}$ (interference forwarding), on the OBRC. Then, the following conditions are easily seen to provide an achievable region

$$R_{1p} \leq \mathcal{C}(P_1) \quad (14a)$$

$$R_{2c''} + R_{1p} \leq \mathcal{C}(P_1 + a_{21}^2P_2) \quad (14b)$$

$$R_{2c} \leq \mathcal{C}\left(\frac{P_2}{1 + a_{12}^2P_1}\right) \quad (14c)$$

$$R_{1R} \leq \eta_{MAC}\mathcal{C}(b_1^2P_{1R}) \quad (14d)$$

$$R_{2c'} + R_{2R} \leq \eta_{MAC}\mathcal{C}(b_2^2P_{2R}) \quad (14e)$$

$$R_{1R} + R_{2c'} + R_{2R} \leq \eta_{MAC}\mathcal{C}(b_1^2P_{1R} + b_2^2P_{2R}) \quad (14f)$$

$$R_{2c'} + R_{1R} \leq \eta_{BC}\mathcal{C}(c_1^2\xi P_R) \quad (14g)$$

$$R_{2R} \leq \eta_{BC}\mathcal{C}\left(\frac{c_2^2\xi P_R}{1 + c_2^2\xi P_R}\right) \quad (14h)$$

Using Fourier-Motzkin elimination method, with the fact that $R_1 = R_{1p} + R_{1R}$, $R_{2c} = R_{2c'} + R_{2c''}$, and $R_2 = R_{2c} + R_{2R}$, the achievable region in Proposition 3 can be obtained. Now, for $b_2, c_1 \rightarrow \infty$, the achievable region becomes

$$R_1 \leq \mathcal{C}(P_1) + \eta_{MAC}\mathcal{C}(b_1^2P_{1R}) \quad (15)$$

$$R_2 \leq \mathcal{C}\left(\frac{P_2}{1 + a_{12}^2P_1}\right) + \eta_{BC}\mathcal{C}(c_2^2\xi P_R) \quad (16)$$

since the overall region is maximized for $\xi = 0$ for $b_2, c_1 \rightarrow \infty$. The converse of Prop. 3 is again obtained from Prop. 1, similar to Prop. 2.

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